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Book Review

Publishers are invited to send two copies of new books for review to Dr. I. Michael Ross, Code: AA/Ro, Department of Aeronautics and Astronautics, U.S. Naval Postgraduate School, 699 Dyer Road, Monterey, CA 93943.

Nonholonomic Motion of Rigid Mechanical Systems from a DAE Viewpoint

Patrick J. Rabier and Werner C. Rheinboldt, Siam, 2000, 140 pp., $36.00, paperback, ISBN 0-89871-446-X

This is a well-written book. It provides a novel and systematic formulation for constrained three-dimensional rigid-body motion. The end result is a theoretically sound treatment of nonholonomic constraints and differential algebraic equation (DAE) formulations that are numerically effective.

The authors state in their preface that "nothing can really be new that addresses the motion of rigid bodies." Despite this modest observation, one can certainly find some gaps in interpreting the literature of this vast and venerable subject. A notable contribution of this monograph is the case of extending the Gauss' principle of least action for constrained motion of rigid bodies without any pre-conditions on the nature of the external forces. Other important new results presented here relate to existence and uniqueness of solutions to DAE formulations. It is shown that these results essentially hinge upon a so-called full-rank condition. For the most part, all the developments hold for general nonholonomic constraints and they specialize readily for the holonomic case, too.

The notation is clear and consistent. There are a total of nine chapters, each with a clear scope and direction. Some important references are listed at the end of the book. At the end of every chapter, the planar motion special case is presented to illustrate the simplifications that happen starting from the three-dimensional case. The appendix covers the background mathematical definitions and theorems used within the main passages without causing disruption to the reading flow.

Chapter one contains introductory material, which is basically an overview of the remaining chapters of the monograph. It helps the reader's understanding of the scope of the work while providing the historical perspective. It is here that the uninitiated will get to see for the first time the difference between an ODE system and an integrodifferential equation system.

Chapter two, The Gauss Principle for Mass Points, is conventional material dealing with the principle of least action in the context of mass points whose motions are constrained. It is shown for conservative systems that coordinate transformations preserve the DAE structure. This fact does not hold true in the nonconservative case. This is a particularly useful chapter because one begins to feel comfortable with the notation.

Chapter three deals with the configuration of rigid bodies. The topological properties of SO (3) and the numerical difficulties encountered while embedding them in linear spaces are discussed. Euler angles are presented as a possible three-dimensional parameterization for SO (3). The quarternion algebra, derivation of the Euler parameters and Euler's fundamental rotation theorem are interesting because of the approach adopted is more standard in physics literature. However, there is no mention of other possible parameterizations for SO (3). For example, the Gibbs vector and modified Rodrigues parameters are not dealt with in this book.

Chapter four, Unconstrained Rigid Body Motions, develops a second-order ODE representation on $\mathcal{R}^3 \times \mathcal{S}^3$ in contrast to the more traditional (and abstract) constructs on $\mathcal{R}^3 \times SO(3)$. In the words of the authors: "Our approach here bypasses the use of Lie group theory and leads to a simple second order ODE." This second-order ODE representation proves to be quite handy for extension to the constrained motion case as a DAE.

In chapter five, the second-order ODE formulations are extended to the constrained motion case. Initially the constrained motion of a single rigid body is considered. Later the equations for a system of rigid bodies subject to constraints are developed. The generalized Gauss' principle forms the bedrock of these discussions in a way that the introduction of Lagrange multipliers makes sense. In many ways, this chapter is the heart of the monograph.

Chapters six and seven cover some important mathematical groundwork dealing with existence and uniqueness of solutions for DAE systems derived in chapter five. While being very technical in content, they could also be bypassed by someone primarily interested in simple DAE system formulations.

Chapter eight presents the practical and application-related aspects of the theory developed in this book. Notably, it gives an algorithm for solving a broad class of DAE systems that incorporate nonholonomic constraints. There is also a broad discussion on costraint drift suppression techniques within this context.

The last chapter is important and presents computational results for several examples that demonstrate the applicability of the DAE solver algorithms and also their effectiveness. The examples are "ramped" to range from simple planar problems with known exact solutions to complex three-dimensional systems with rolling balls over inclined surfaces.

This is a wonderful book providing a contemporary treatment of the subject. It should be a valuable addition to the personal library of any engineer with a mathematical bent of mind. Although written in a monograph format, it is also highly suitable for an advanced graduate level (elective) course in multibody dynamical systems subject to nonholonomic constraints.

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